Detector Physics

Radiation Interacting with Matter

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Outline

- Radiation Interacting with Matter
  - Cross Section (σ)
  - Mean Free Path (λ)
  - Energy Loss
    - Electrons
    - Cherenkov Radiation
    - Bremsstrahlung
    - Phontons
    - Neutrons

Charged particles going through matter

- Two things happen most of the time:
  - Inelastic collisions with the target's electrons
  - Elastic scattering of the target's nuclei
- More interesting things happen less often
  - Cherenkov radiation emission
  - Nuclear reactions
  - Bremsstrahlung

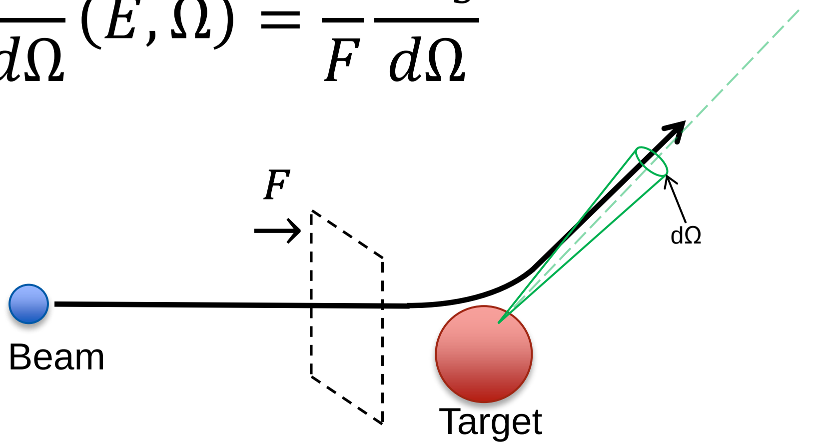
The reason we're in business

$$1 \text{ barn} = 10^{-24} \text{ cm}^2$$

Idealized Cross Section (σ) – 1 scatterer

- Probability per unit solid angle that the beam will scatter.
  - Unit analysis forces $d\sigma$ to be in units of area
  - <u>**Not really a dimension**</u>
- Total cross section σ(E)
  - Integrated over all solid angles

$$\frac{d\sigma}{d\Omega}(E,\Omega) = \frac{1}{F}\frac{dN_s}{d\Omega}$$



$$\sigma(E) = \int d\Omega \frac{d\sigma(E,\Omega)}{d\Omega}$$

Thin sheet of material

- Assume a large, thin target
  - $A_{\mathrm{tgt}} > A_{\mathrm{beam}} = A$
  - Thickness = δx
  - Density of scatterers = n
  - $P(x)$ = Prob. of having interaction by x *in material*.
    - $P(0) = 0$
    - $P(\infty) = 1$

$$N_s(\Omega) = FAn\delta x \frac{d\sigma}{d\Omega}$$

$$N_{tot} = FAn\sigma \cdot \delta x$$

$$P(\delta x) = n\sigma \cdot \delta x$$

Interaction probabilities

$$P(x + \delta x) = P(x) + \underbrace{[1 - P(x)]}_{\text{}} n\sigma\delta x$$

The probability of
not interacting

Interaction probabilities

$$P(x+\delta x) = P(x) + [1 - P(x)]n\sigma\delta x$$

$$\frac{P(x+\delta x) - P(x)}{\delta x} = [1 - P(x)]n\sigma$$

Definition of a derivative

Interaction probabilities

$$P(x+\delta x) = P(x) + [1 - P(x)]n\sigma\delta x$$

$$\frac{P(x+\delta x) - P(x)}{\delta x} = [1 - P(x)]n\sigma$$

$$\frac{dP}{dx} = [1 - P]n\sigma$$

Interaction probabilities

$$P(x+\delta x) = P(x) + [1 - P(x)]n\sigma\delta x$$

$$\frac{P(x+\delta x) - P(x)}{\delta x} = [1 - P(x)]n\sigma$$

$$\frac{dP}{dx} = [1 - P]n\sigma$$

$$\frac{d[1-P]}{dx} = -[1 - P]n\sigma$$

- The change in the probability of noninteraction is *necessarily* the negative of the change in the probability of interaction.

Interaction probabilities

$$P(x+\delta x) = P(x) + [1-P(x)]n\sigma\delta x$$

$$\frac{P(x+\delta x) - P(x)}{\delta x} = [1-P(x)]n\sigma$$

$$\frac{dP}{dx} = [1-P]n\sigma$$

$$\frac{d[1-P]}{dx} = -[1-P]n\sigma$$

$$[1-P] = C_0 e^{-n\sigma x}$$

- From the initial conditions, $P(0) = 0$ so $C_0 = 1$

Interaction probabilities

Prob. of interaction between 0 an x

$$P(x) = 1 - e^{-n\sigma x}$$

$$1 - P(x) = e^{-n\sigma x}$$

Prob. of not interacting between 0 and x

Mean free path - λ

- Given a $P(x)$ we can get $W(x)$ – the Probability density function
  - As long as the cumulative distribution function of $P(x)$ is continuous.
- From there, λ is just the expectation value of x

$$W(x) = \frac{d}{dx}P(x) = \frac{d}{dx}\left(1 - e^{-n\sigma x}\right) = n\sigma e^{-n\sigma x}$$

$$\lambda = \int_0^{\infty} xW(x)dx = \int_0^{\infty} xn\sigma e^{-n\sigma x}dx$$

Mean free path - λ

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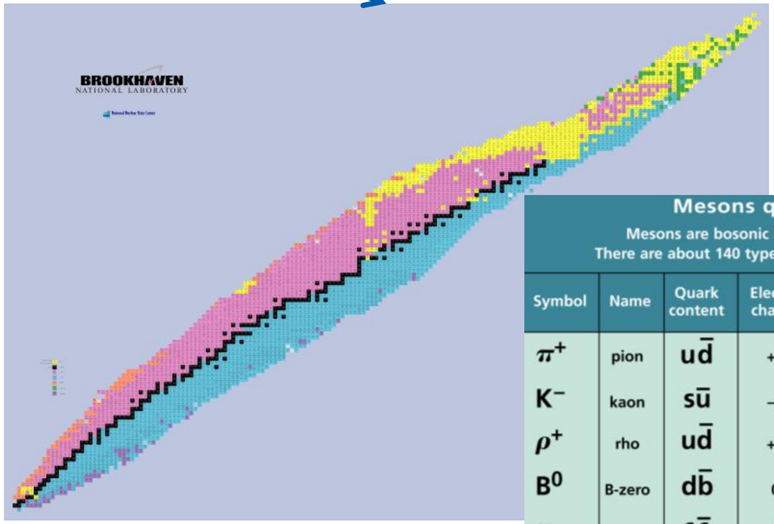
$$W(x) = \frac{d}{dx}P(x)$$
$$= \frac{d}{dx}(1 - e^{-n\sigma x})$$
$$= n\sigma e^{-n\sigma x}$$

$$\lambda = \int_0^\infty xW(x)dx$$
$$= \int_0^\infty xn\sigma e^{-n\sigma x}dx$$
$$= \frac{1}{n\sigma}$$

Charged particle energy loss

Dealing with 2 regimes

e^- & e^+



Mesons $q\bar{q}$

Mesons are bosonic hadrons.
There are about 140 types of mesons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
π^+	pion	$u\bar{d}$	+1	0.140	0
K^-	kaon	$s\bar{u}$	−1	0.494	0
ρ^+	rho	$u\bar{d}$	+1	0.770	1
B^0	B-zero	$d\bar{b}$	0	5.279	0
η_c	eta-c	$c\bar{c}$	0	2 .980	0

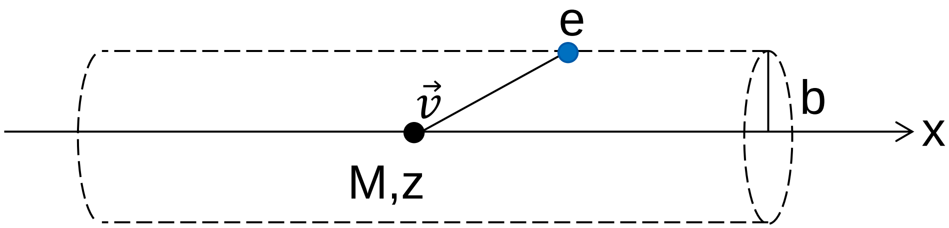
Just about everything else

Let's look at the classical picture

Heavy Particle Passing Through Material

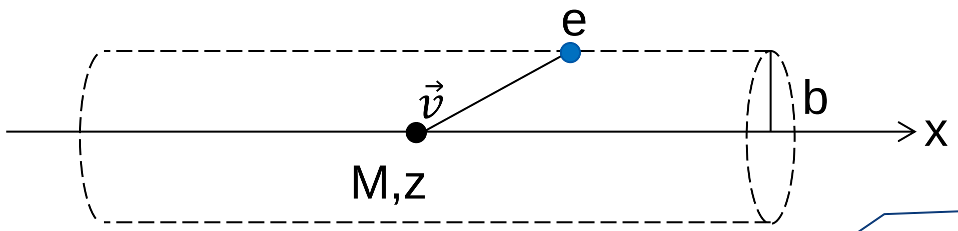
- Heavy Particle:
  - Mass – $M \gg m_e$
  - Charge – z
  - Velocity – $\beta = \frac{v}{c}$
  - Negligible deviation from path
- Electron is:
  - Free
  - Initially at rest
  - Small movement due to interaction

Let's look at the classical picture



$$I = \int F\, dt = e \int E_{\perp} dt =$$

Let’s look at the classical picture



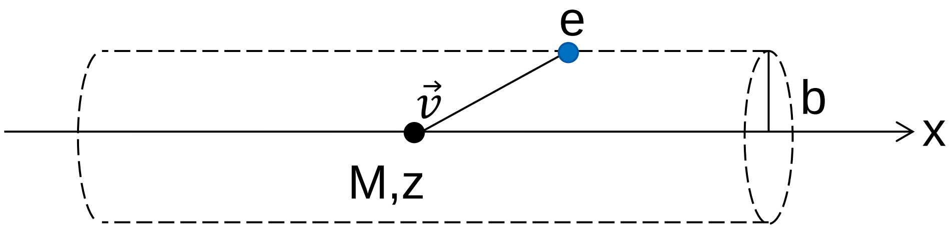
$$I = \int F\,dt = e\int E_{\perp} dt =$$

$$= e\int E_{\perp}\frac{dt}{dx}dx = e\int \frac{E_{\perp}}{v}dx$$

All other components of E cancel out

v is not a function of x

Let's look at the classical picture



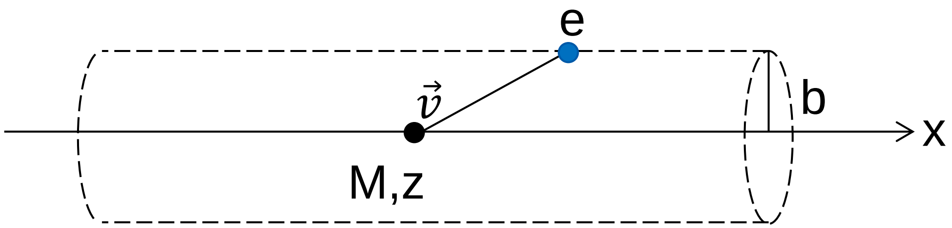
$$I = \int F\, dt = e \int E_\perp dt =$$

$$= e \int E_\perp \frac{dt}{dx} dx = \frac{e}{v} \int E_\perp dx$$

Apply Here

$$\int E_\perp 2\pi b\, dx = 4\pi z e \rightarrow \int E_\perp dx = \frac{2ze}{b}$$

Let's look at the classical picture



$$I = \int F\,dt = e\int E_{\perp}dt =$$

$$= e\int E_{\perp}\frac{dt}{dx}dx = e\int \frac{E_{\perp}}{v}dx$$

$$= \boxed{\frac{2ze^2}{b}}$$

$$\int E_{\perp}2\pi b\,dx = 4\pi ze \rightarrow \int E_{\perp}dx = \frac{2ze}{b}$$

The energy gained by the e$^-$ is:

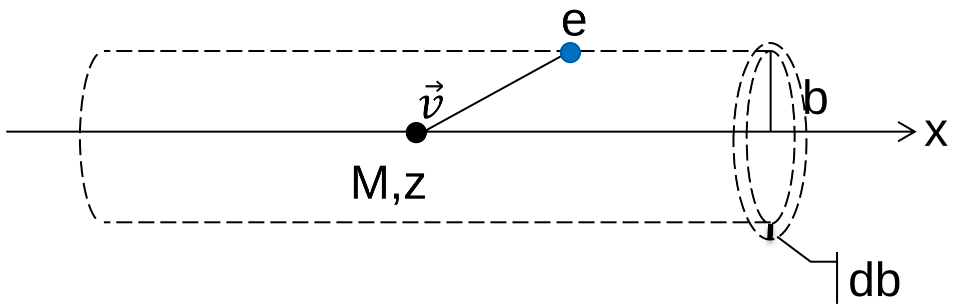
$$I = \frac{2ze^2}{bv}$$

$$\Delta E(b) = \frac{p^2}{2m} = \frac{I^2}{2m_e}$$

$$= \frac{2z^2e^4}{b^2v^2m_e}$$

So the energy lost by the heavy ion is:

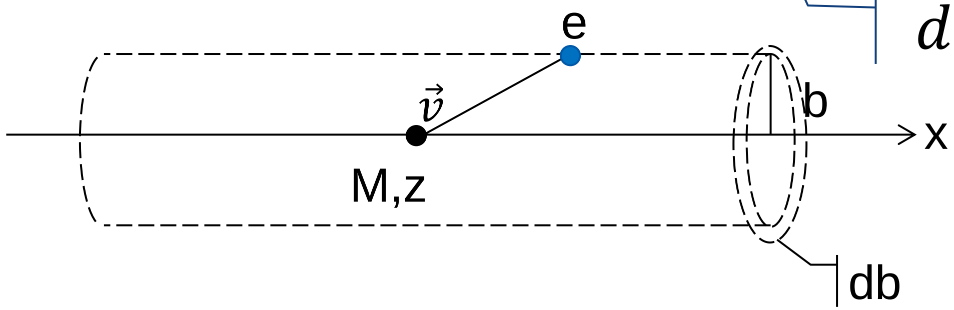
$$-dE(b) = \Delta E(b) n_e \, dV$$



So the energy lost by the heavy ion is:

$$-dE(b) = \Delta E(b) n_e \, dV$$
$$= \frac{2z^2e^4}{b^2v^2m_e} n_e \, dV$$

$$dV = 2\pi b \, db \, dx$$



So the energy lost by the heavy ion is:

$$
\begin{aligned}
-dE(b) &= \Delta E(b) n_e \, dV \\
&= \frac{2z^2e^4}{b^2v^2m_e} n_e \, dV \\
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&= \frac{4\pi z^2 e^4 n_e}{v^2 m_e} \frac{db}{b} dx
\end{aligned}
$$

- Complications:
  - Can't integrate over b=0
  - Have to integrate from $b_{\text{min}} \rightarrow b_{\text{max}}$

So the energy <u>lost</u> by the heavy ion is:

$$
\begin{aligned}
-dE(b) &= \Delta E(b) n_e \, dV \\
&= \frac{2z^2e^4}{b^2v^2m_e} n_e \, dV \\
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&= \frac{4\pi z^2 e^4 n_e}{v^2 m_e} \frac{db}{b} dx \\
-\frac{dE}{dx} &= \frac{4\pi z^2 e^4 n_e}{v^2 m_e} \ln \frac{b_{\max}}{b_{\min}}
\end{aligned}
$$

b_{max} & b_{min}

b_{min}

- Head on collision
  - e⁻ gets $\Delta E_{\mathrm{max}} = 2\gamma^2 m_e v^2$

$$\frac{2z^2e^4}{m_e v^2 b_{\mathrm{min}}^2} = 2\gamma^2 m_e v^2$$

$$b_{\mathrm{min}} = \frac{ze^2}{\gamma m_e v^2}$$

b_{max}

- Assume bound, orbiting electron
  - $t_{\mathrm{int}} \leq \tau = \frac{1}{\nu}$

Interaction time

Frequency, not velocity

$$t = \frac{b}{v} \rightarrow \frac{b}{\gamma v}$$

$b_{\max}$ & $b_{\min}$

$b_{\min}$

- Head on collision
  - e⁻ gets $\Delta E_{\max} = 2\gamma^2 m_e v^2$

$$\frac{2z^2e^4}{m_e v^2 b_{\min}^2} = 2\gamma^2 m_e v^2$$

$$b_{\min} = \frac{ze^2}{\gamma m_e v^2}$$

$b_{\max}$

- Assume bound, orbiting electron
  - $t_{\text{int}} \leq \tau = \frac{1}{\nu}$

$$t = \frac{b}{v} \rightarrow \frac{b}{\gamma v} \leq \tau = \frac{1}{\bar{\nu}}$$

Avg. orbital freq. over all orbits

b_{max} & b_{min}

b_{min}

- Head on collision
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$$\frac{2z^2e^4}{m_e v^2 b_{\mathrm{min}}^2} = 2\gamma^2 m_e v^2$$

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b_{max}

- Assume bound, orbiting electron
  - $t_{\mathrm{int}} \leq \tau = \frac{1}{\nu}$

$$t = \frac{b}{v} \rightarrow \frac{b}{\gamma v} \leq \tau = \frac{1}{\bar{\nu}}$$

$$b_{\mathrm{max}} = \frac{\gamma v}{\bar{\nu}}$$

So the energy lost by the heavy ion is:

$$
\begin{aligned}
-dE(b) &= \Delta E(b) n_e \, dV \\
&= \frac{2z^2 e^4}{b^2 v^2 m_e} n_e \, dV \\
&= \frac{2z^2 e^4}{b^2 v^2 m_e} n_e 2\pi b \, db \, dx \\
&= \frac{4\pi z^2 e^4 n_e}{v^2 m_e} \frac{db}{b} dx \\
-\frac{dE}{dx} &= \frac{4\pi z^2 e^4 n_e}{v^2 m_e} \ln \frac{b_{\max}}{b_{\min}}
\end{aligned}
$$

$$
-\frac{dE}{dx} = \frac{4\pi z^2 e^4 n_e}{v^2 m_e} \ln \frac{\gamma^2 m_e v^3}{z e^2 \bar{\nu}}
$$

Charged particle energy loss – Due to e$^-$

The Bethe-Block Equation for linear energy transfer

$$-\frac{dE}{dx} = 2\pi N_A r_e^2 m_e c^2 \rho \left(\frac{Z}{A}\right)\frac{z^2}{\beta^2}\left[\ln\left(\frac{2m_e\gamma^2c^2\beta^2W_{\max}}{I^2}\right) - 2\beta^2 - \delta(\beta) - 2\frac{C}{Z}\right]$$

- Energy loss/unit length: $-\frac{dE}{dx}$
- Target: $\left(\frac{Z}{A}\right)$
- Beam: z^2
- Max E transfer to e$^-$: $W_{\max}$
- Mean excitation energy (eV): I
- Density correction: $\delta(\beta)$
- Shell Correction: C

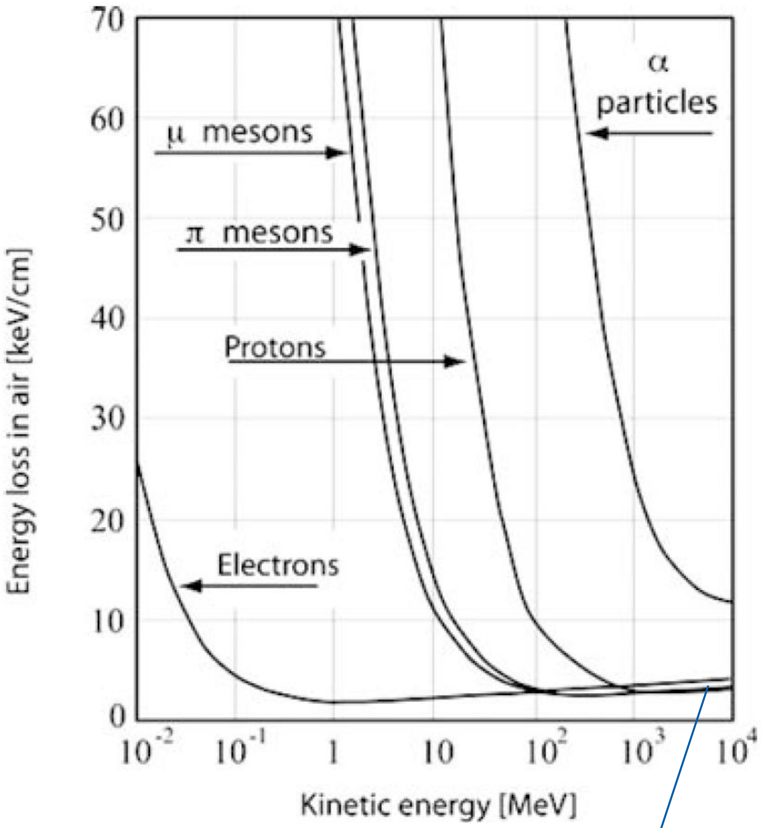
$$W_{\max} = \frac{2m_e(v\gamma)^2}{1 + 2\frac{m_e}{M}\sqrt{1+(\beta\gamma)^2} + \left(\frac{m_e}{M}\right)^2}$$

$$\simeq 2m_e(v\gamma)^2$$

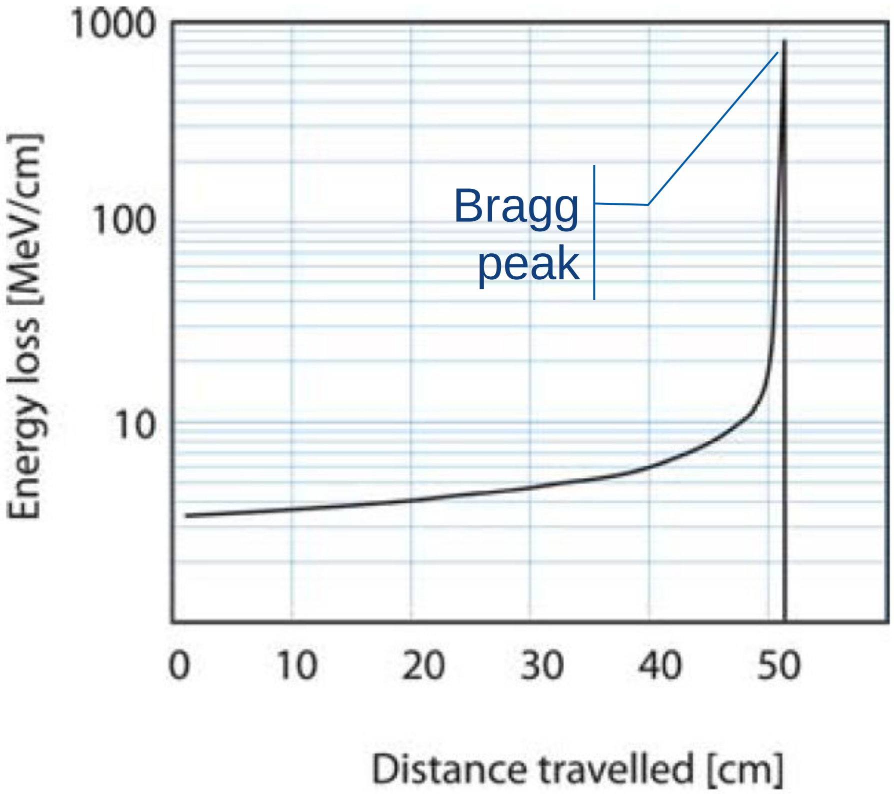
If $M \gg m_e$

C. Leroy and P. Rancoita, *Principles of radiation interaction in matter and detection*, World Scientific (2004).

Charged particle energy loss – Due to e⁻



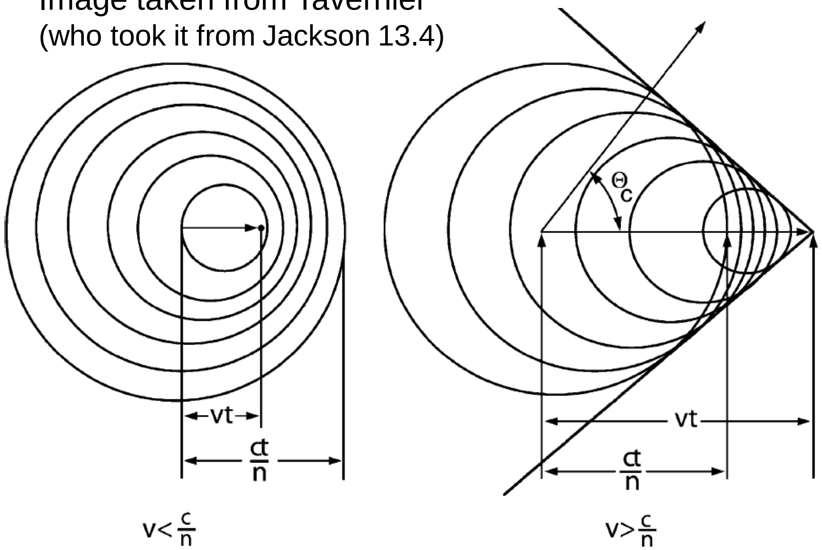
Electrons don't have much mass



Range: The distance a particle travels in medium before coming to rest

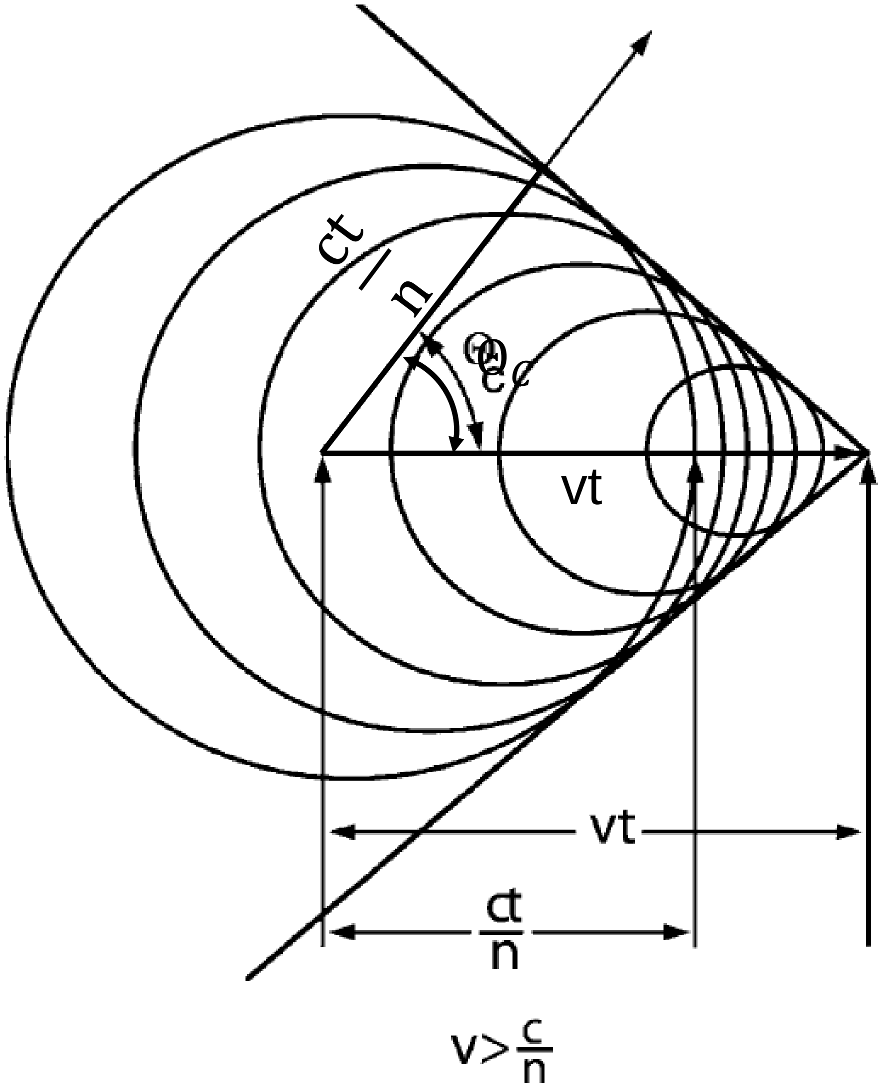
Cherenkov Radiation

Image taken from Tavernier
(who took it from Jackson 13.4)



Material	n
Vacuum	1.00000000000000000000000000000000
Air (@ STP)	1.00028
Water (@ 20° C)	1.333
Sodium Iodide	1.736
Silicon	3.443
Germanium	4.065

Cherenkov Radiation



$$\cos(\Theta_c) = \frac{\left(\frac{ct}{n}\right)}{vt} = \frac{c}{nv}$$

$$E_{\min} = mc^2\left(\sqrt{\frac{n^2}{n^2-1}} - 1\right)$$

Material	Neutron (MeV)	Electron (keV)
Air (@ STP)	38,772	21
Water (@ 20° C)	481	0.262
Sodium Iodide	209	0.114
Silicon	42	0.023
Germanium	29	0.016

Bremsstrahlung

- Emitted whenever a charged particle is accelerated
  - Collisions
  - B&E Field deflections

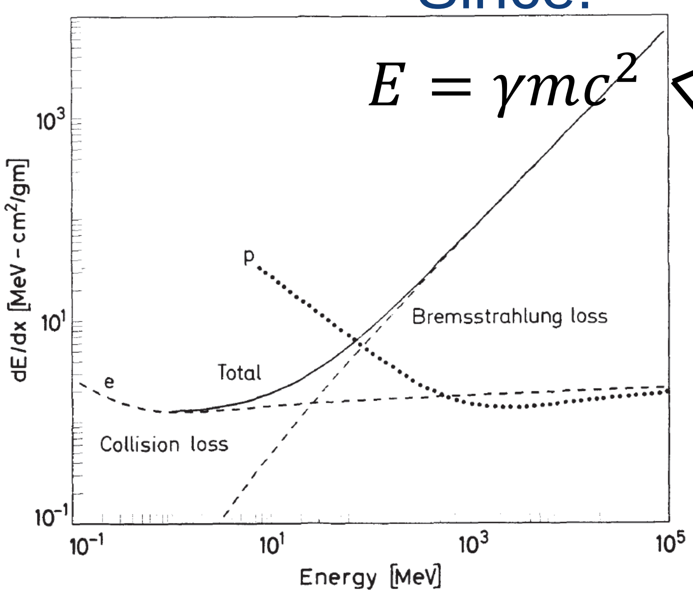
Total radiated power:

$$P = \frac{e^2\gamma^6}{6\pi\varepsilon_0 c}\left[\dot{\vec{\beta}}^2 - \left(\vec{\beta}\times\dot{\vec{\beta}}\right)^2\right]$$

Linear acceleration:

Since:

$$E = \gamma m c^2$$



$$P \sim m^{-6} \Longrightarrow \quad P = \frac{e^2\gamma^6}{6\pi\varepsilon_0 c}\dot{\vec{\beta}}^2$$

Perpendicular acceleration:

$$P \sim m^{-4} \Longrightarrow \quad P = \frac{e^2\gamma^4}{6\pi\varepsilon_0 c}\dot{\vec{\beta}}^2$$

Charge Particle Overview (100 keV – 10 MeV)

	Interaction Description
α's	~1000 MeV/cm/(mass density). Range ~10 μm in solid and cm in gas. Straight trajectory
β's	~2 MeV/cm/(charge density). Significant loss due to bremsstrahlung. Multiple scattering creates "random walk" trajectory. When β^+ stop, they annihilate -> 2 x 511 keV γ
Protons	Range ~ 1 mm in solid and ~1 m in gas. Straight trajectory.
Nuclear fragments	Similar to α's but more massive so more energy loss/distance. ~1 μm in solid. Straight line trajectory.

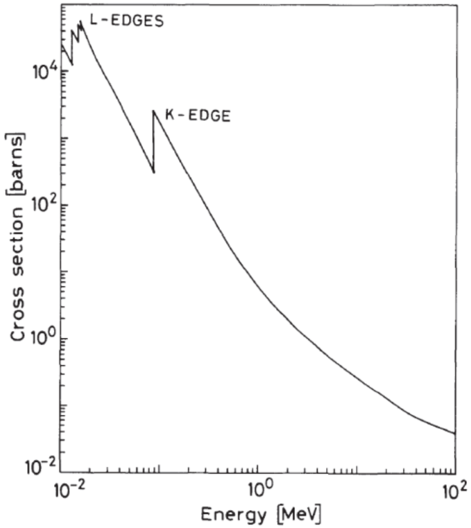
Photons Interacting with Matter

- Photons primarily interact with matter in 3 ways:
  1. Photoelectric Effect
  2. Compton Scattering
  3. Pair Production
  - Some others as well:
    - Photodisintegration: (γ,n), (γ,p), (γ,α)
- Longer range in matter
- Photons experience no energy loss (**absorbed or not**)
  - But beams can be attenuated: $I(x) = I_0 e^{-\mu x}$

The Photoelectric Effect

$$E_{\text{Kinetic}} = \hbar\omega - E_{\text{Binding}}$$

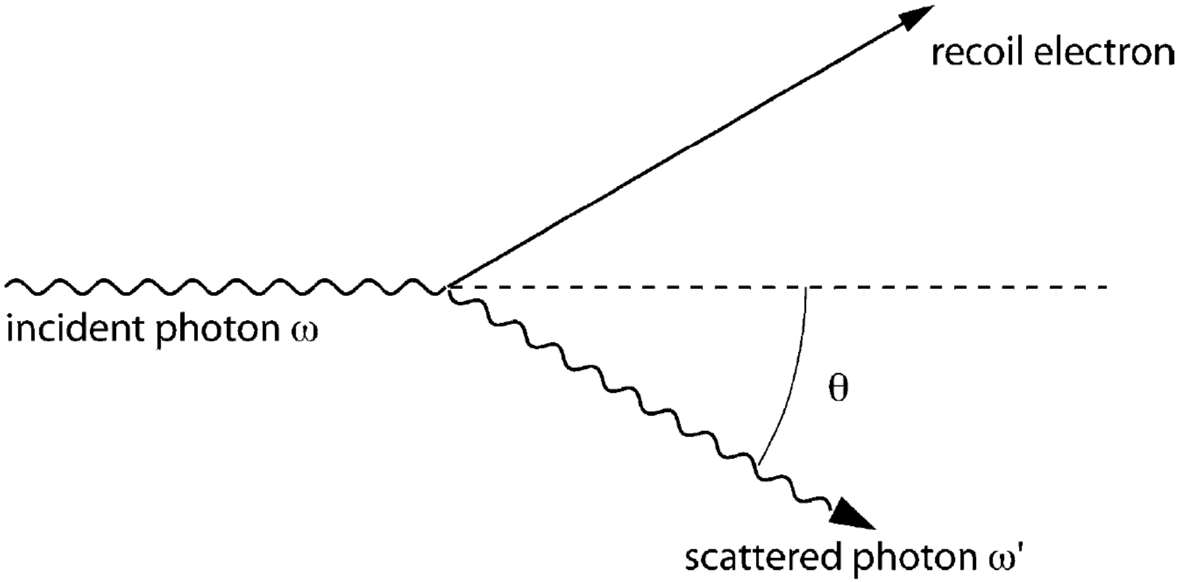
- The photon is totally absorbed.
- If
  - $\hbar\omega > E_{\text{Binding}}$ - Free electron
  - $\hbar\omega < E_{\text{Binding}}$ - Excited, bound electron
- Dominates for $E_\gamma \leq 100\ \text{keV}$
- $\sigma \sim \frac{Z^n}{E_\gamma^{3.5}}$



Compton Scattering - Elastic collision between γ & e^-

$$\hbar\omega' = \frac{\hbar\omega}{\left[1 + \frac{\hbar\omega}{mc^2}(1 - \cos\theta)\right]}$$

For homework, derive

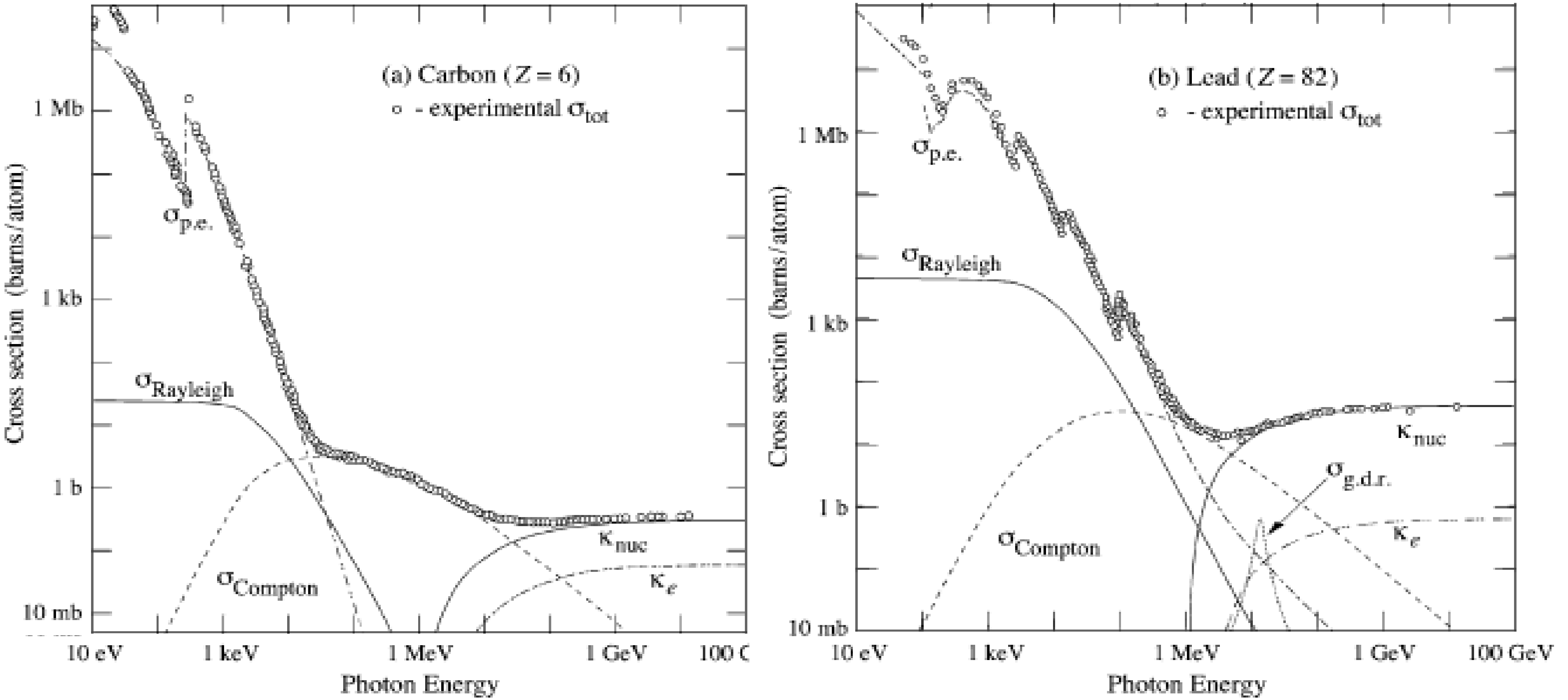


$$\frac{d\sigma}{d\Omega} = \frac{r_e^2}{2}\frac{1}{[1+\gamma(1-\cos\theta)]^2}\left(1 + \cos^2\theta + \frac{\gamma^2(1-\cos\theta)^2}{1+\gamma(1-\cos\theta)}\right)$$

Pair Production

- When $E_\gamma > m_{e^-} + m_{e^+} = 1022$ keV
  - $m_{\mu^-} + m_{\mu^+} = 211{,}316$ keV
  - $m_p + m_{\bar{p}} = 1{,}876{,}544$ keV
- For the production to last, it must occur near the nucleus.
  - Otherwise, momentum would not be conserved

Complete Photon Interaction Range



Neutrons → Strong Interaction only

Slow < 0.5 <u>eV</u>

- Elastic Scattering
- Neutron capture followed by:
  - (n,p)
  - (n,d)
  - (n,α)
  - (n,t)
  - (n,αp)
  - (n,f)

Fission

Fast = 100 keV – 10 MeV

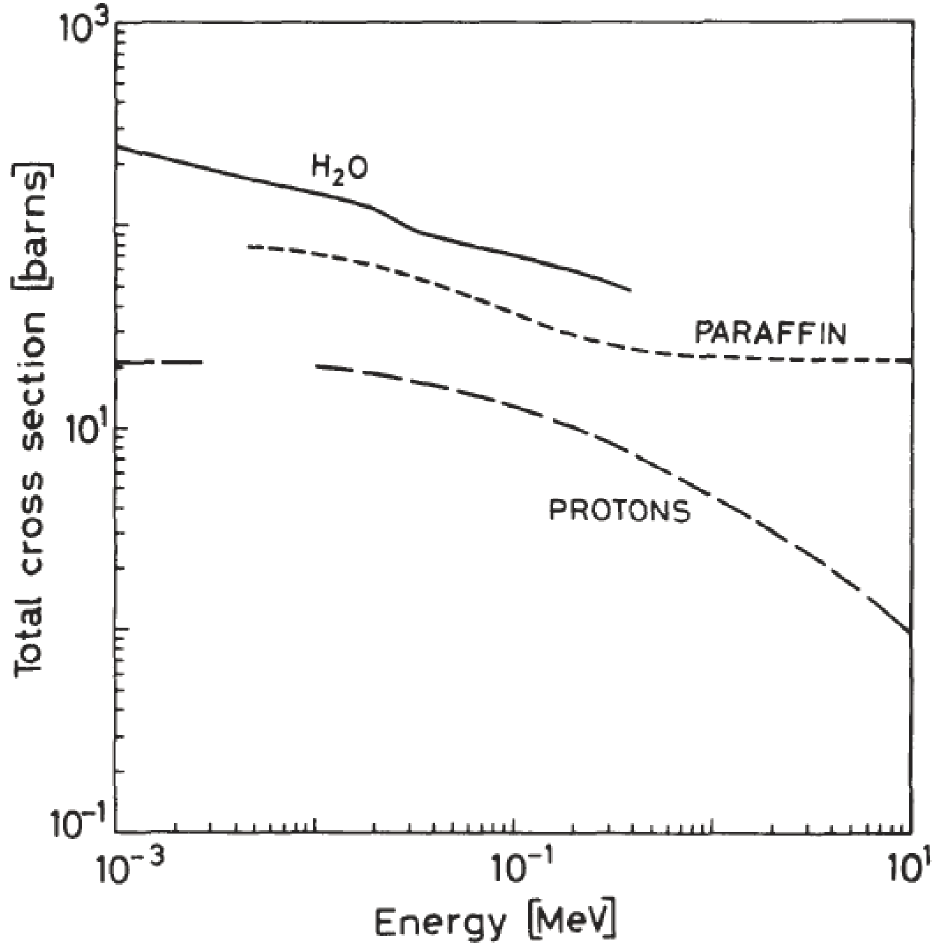
- Elastic Scattering
- Neutron capture followed by….
- Radiative neutron capture (n,γ)
- Inelastic scattering
  - Nucleus is left in an excited state and emits a:
    - Photon
    - Charged particle

Neutrons → Strong Interaction only

High Energy > 100 MeV

- High energy hadronic showers

See: **LHC, the**



Thank you!
Merci

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